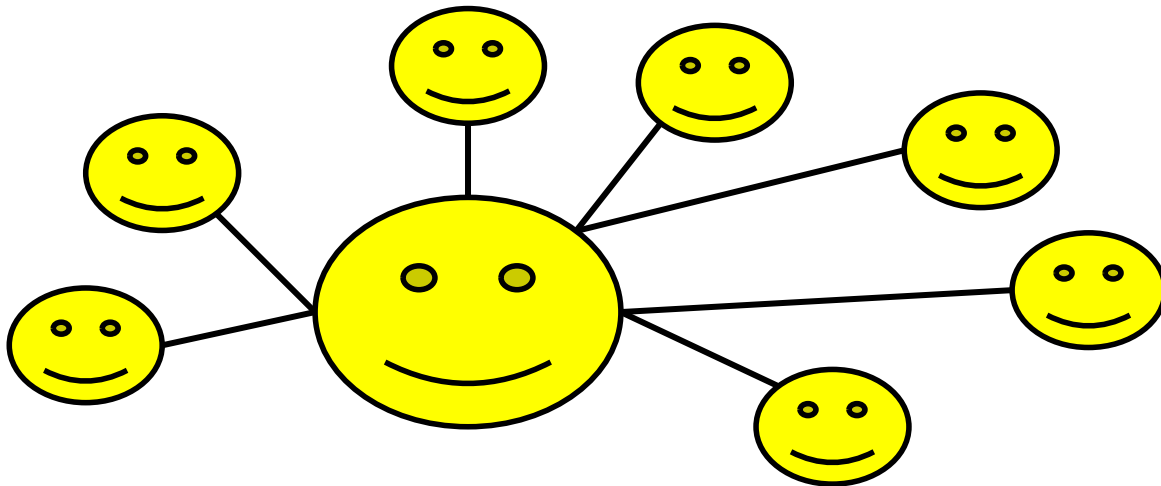




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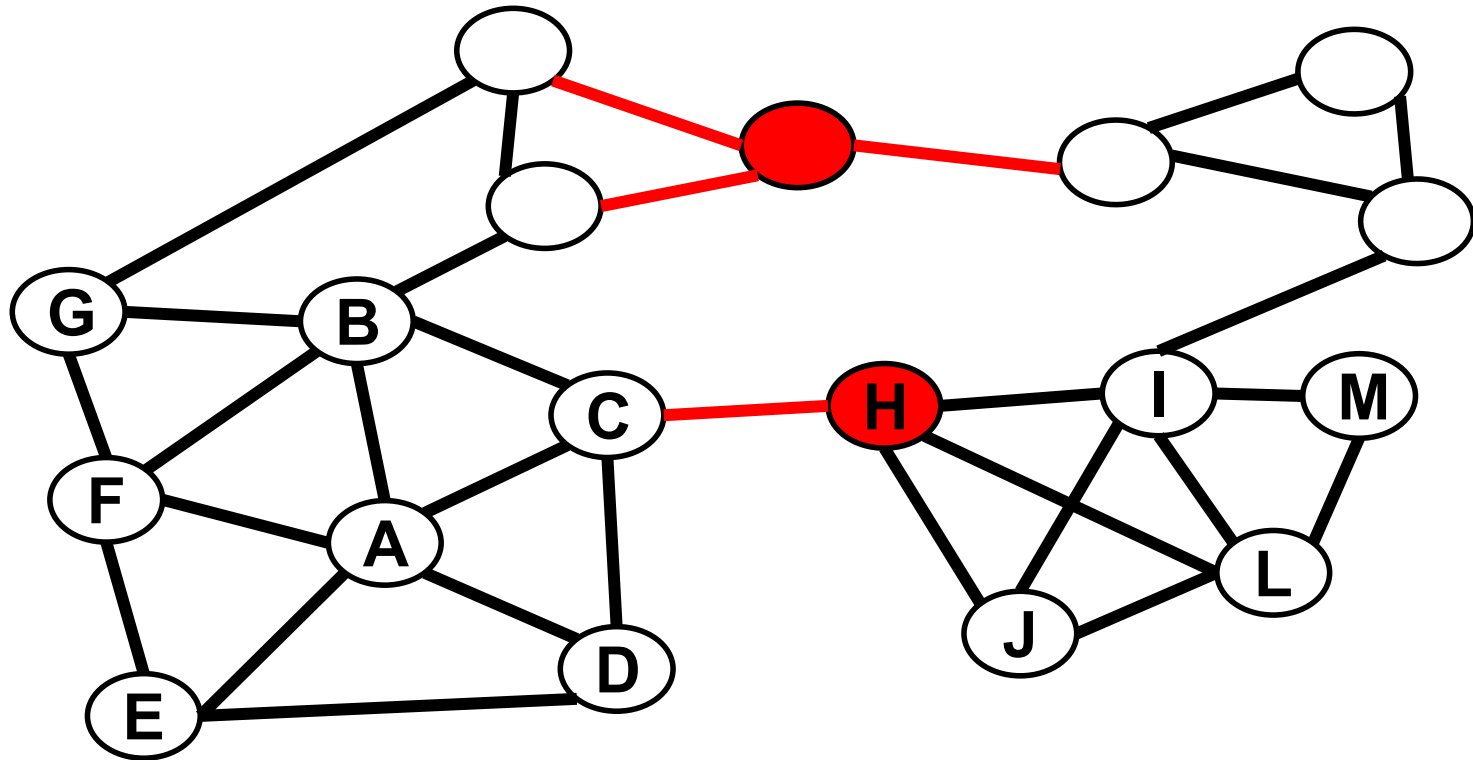


# CENTRALIDADE

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# Importância dos nós



# Importância dos nós



# Importância dos nós



# Centralidade

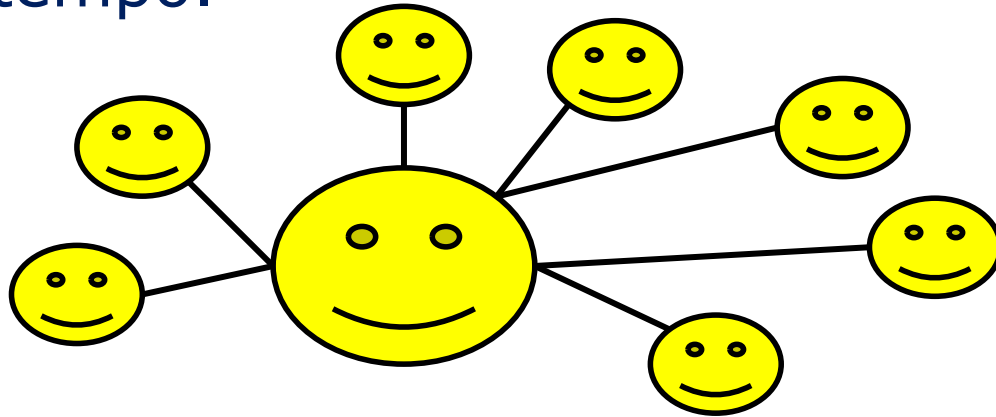
**CENTRALIDADE** indica:

- ❑ Influência de uma pessoa
- ❑ Importância de certas arestas
- ❑ Ordem nos resultados de busca



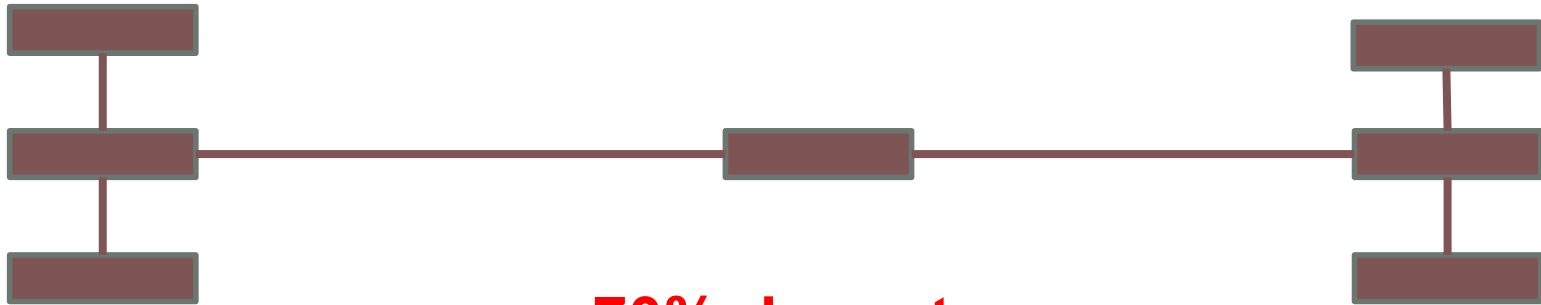
# Exemplos

Em uma **rede social** o nó central é aquele que tem mais conexões, pois influencia mais pessoas ao mesmo tempo.



# Exemplos

**Rede de transporte:** quanto mais pessoas passam por certo ponto.



**70% das rotas  
passam por aqui!!**



# Centralidade

Origem em estudos de sociologia.

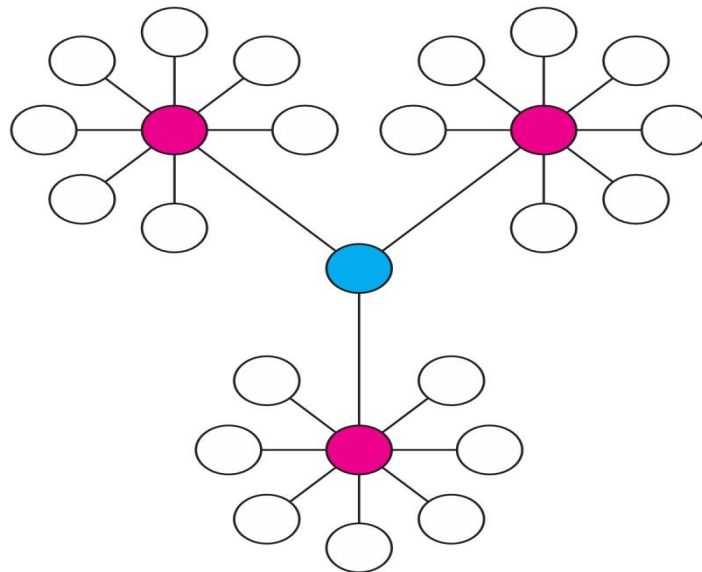
- ❑ Grau
- ❑ Proximidade
- ❑ Betweenness
- ❑ Pagerank (utilizado pelo Google)







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# Grau

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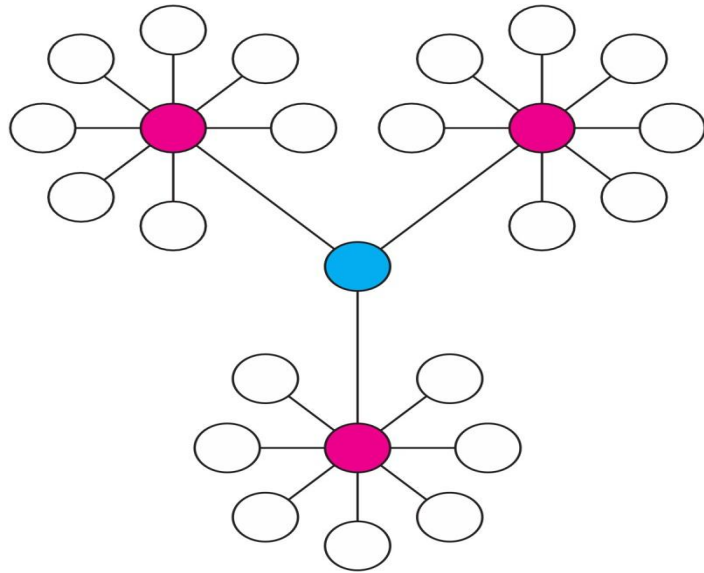
# Centralidade de Grau

A ideia mais simples de um nó central é aquele que está **mais conectado que os demais**, ou seja, aquele que tem um maior grau.

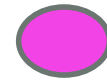
$$C_g(v_i) = \frac{g(v_i)}{|V| - 1}$$



# Centralidade de Grau



$$C_g = 0,042$$



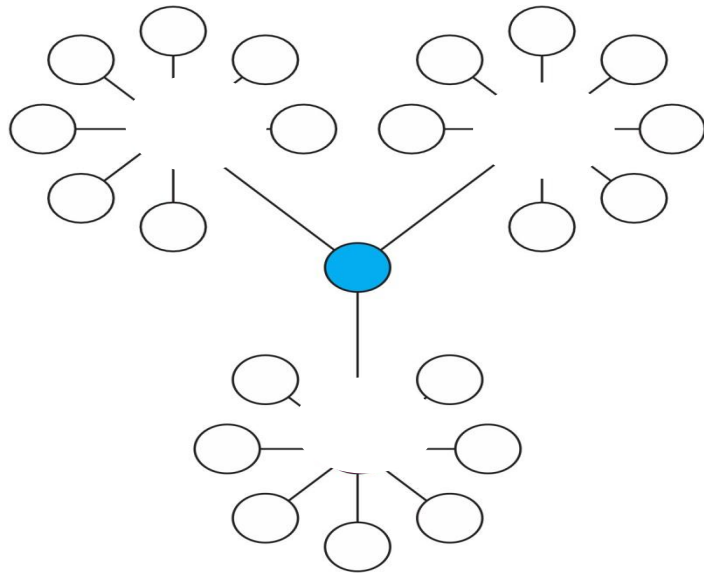
$$C_g = 0,333$$



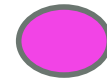
$$C_g = 0,125$$



# Centralidade de Grau



$C_g = 0,042$



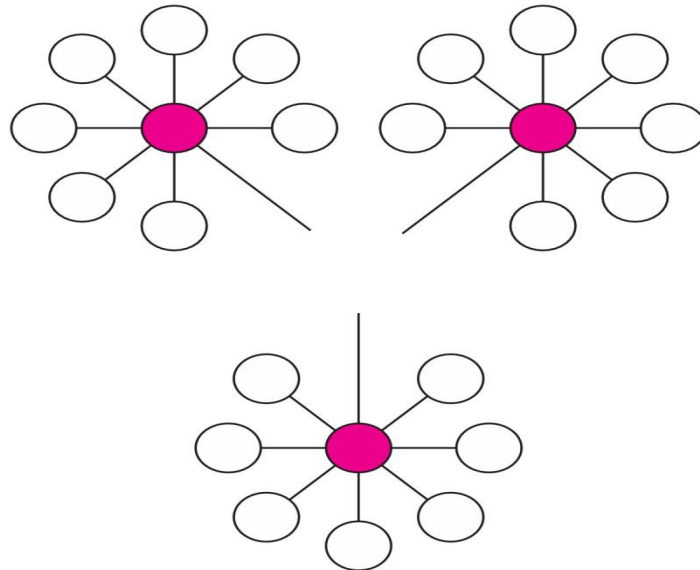
$C_g = 0,333$



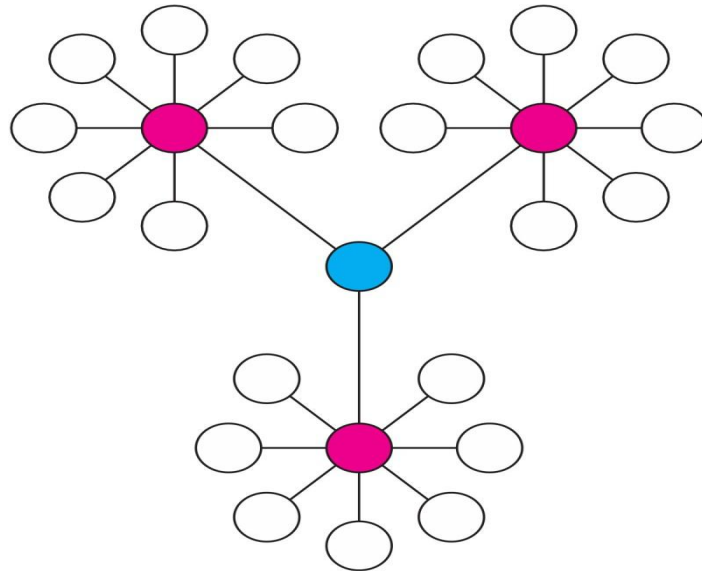
$C_g = 0,125$



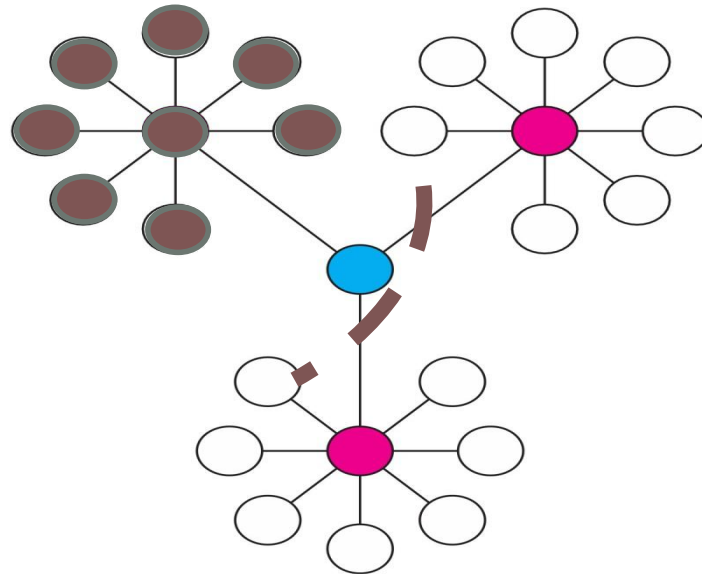
# Considere Isso!



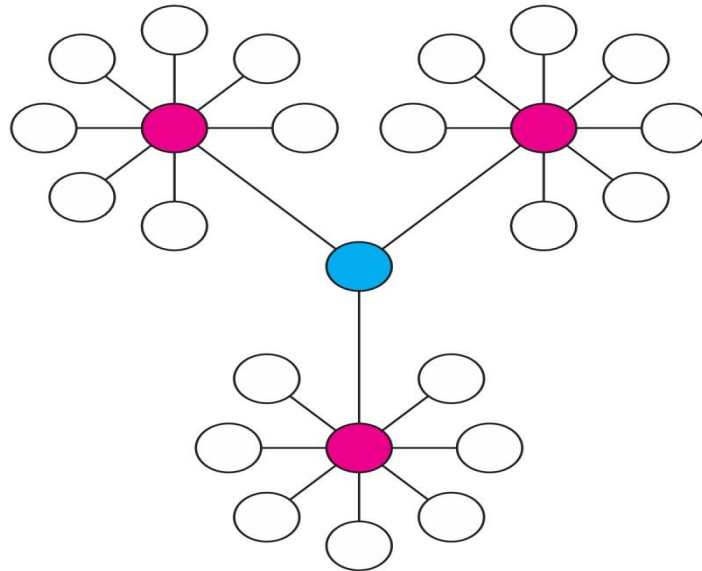
# Considere Isso!



# Considere Isso!

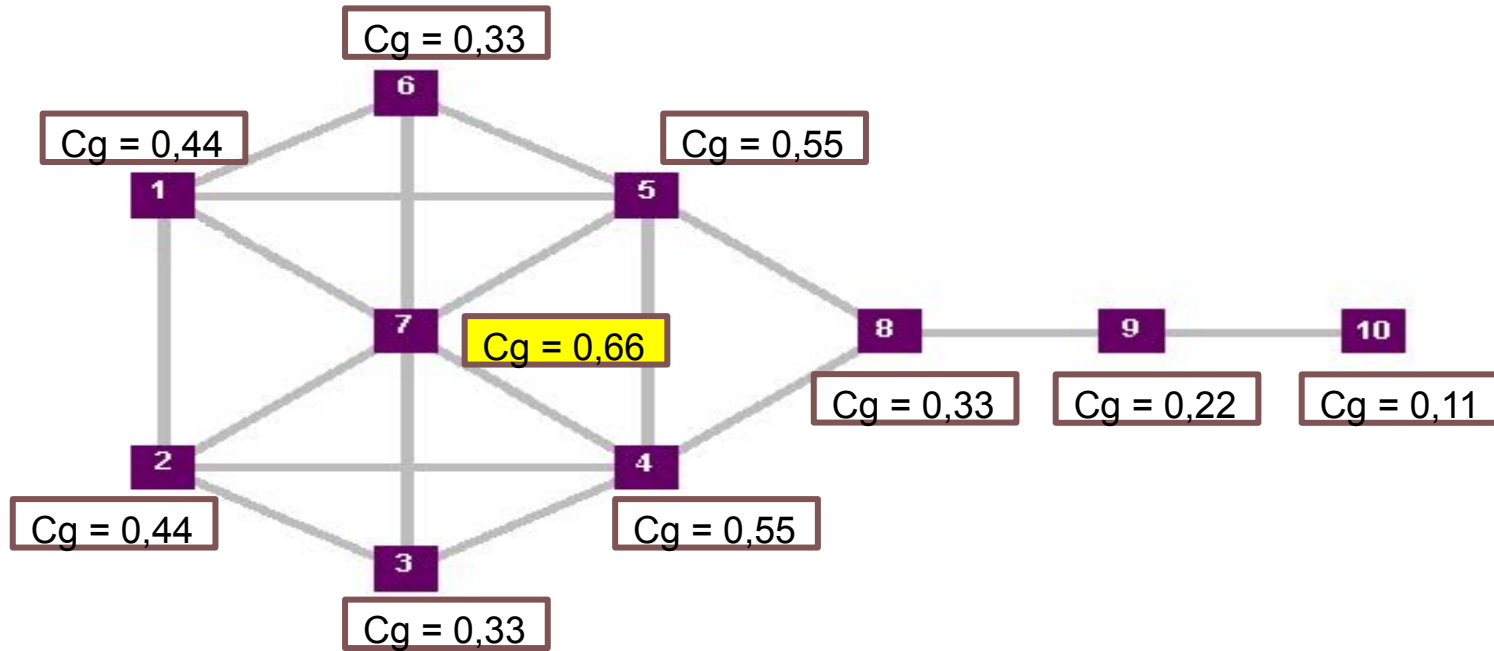


# Considere Isso!

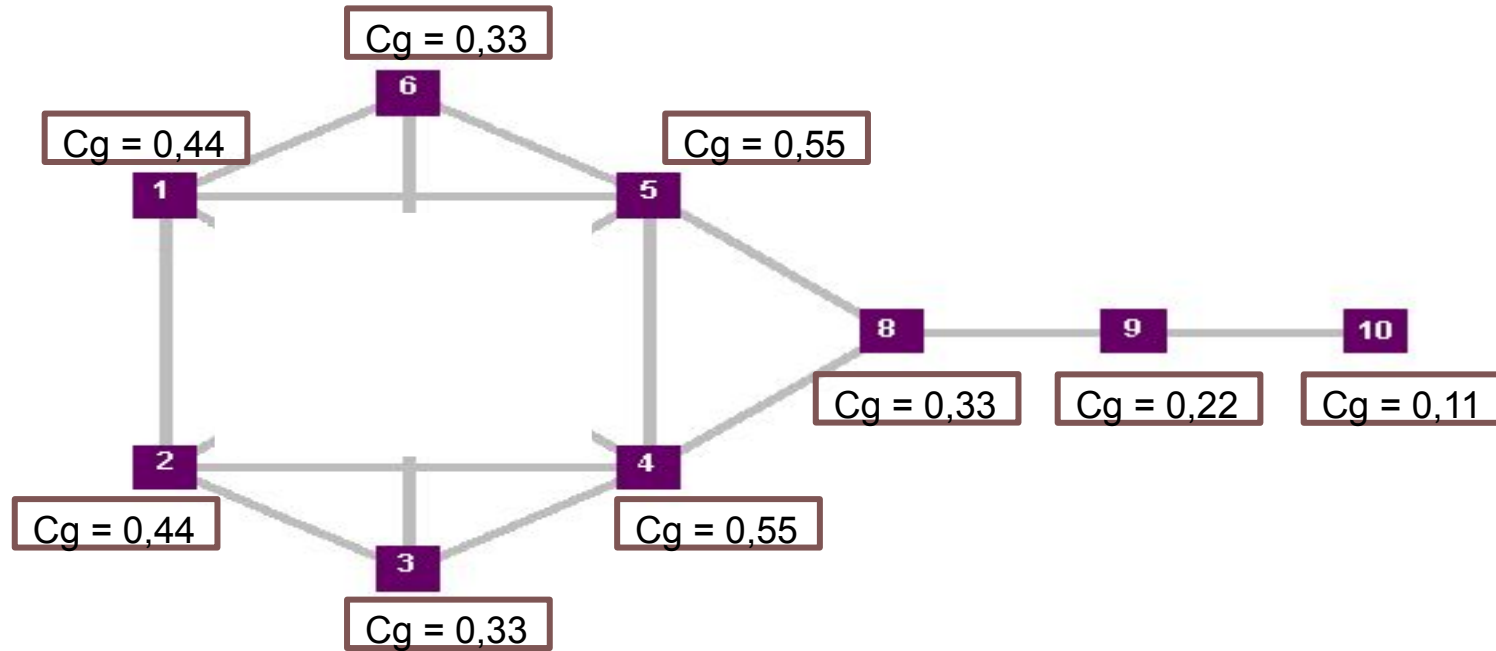




# Centralidade de Grau

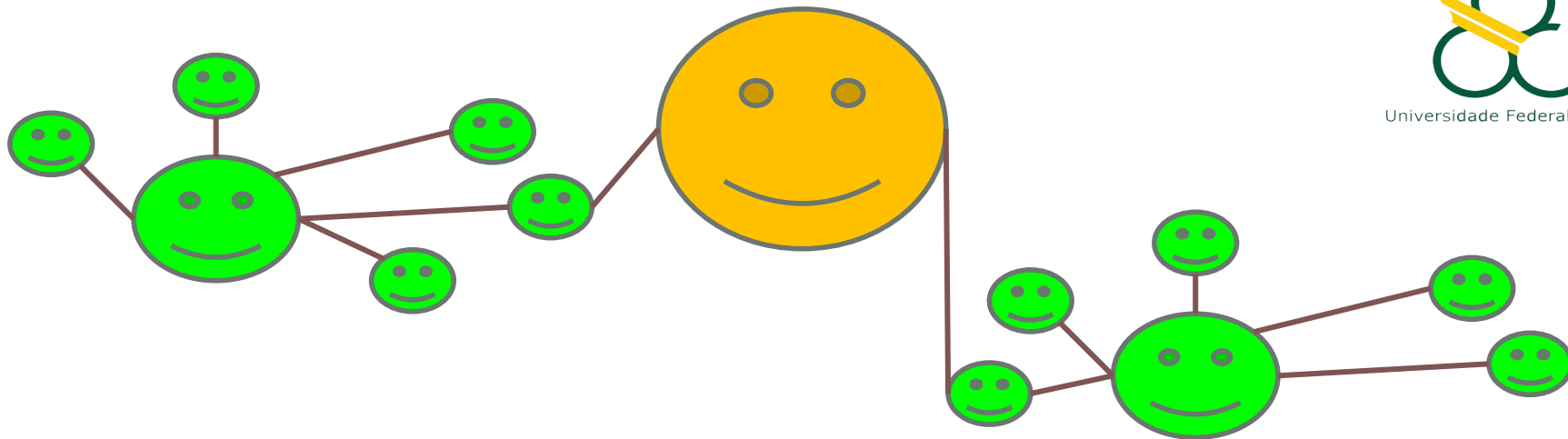


# Centralidade de Grau





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# Proximidade

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# Centralidade de Proximidade



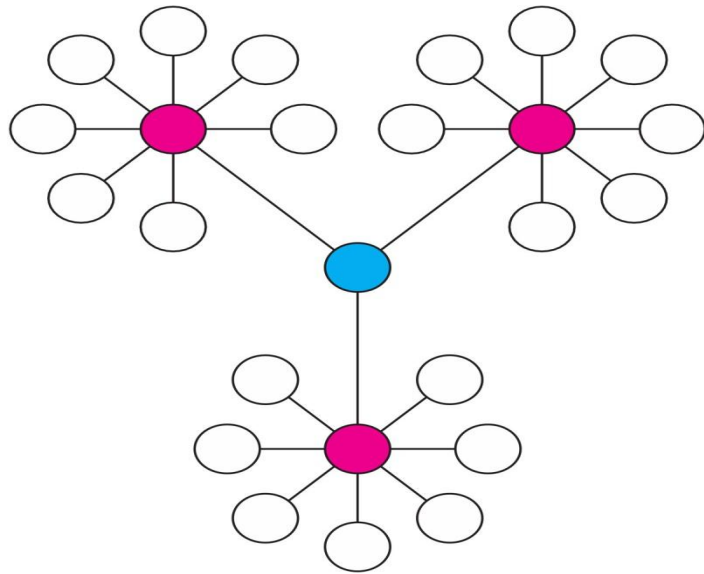
# Centralidade de Proximidade

Inverso da média dos caminhos mais curtos:

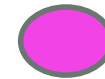
$$C_p(v_i) = \frac{n - 1}{\sum_{v_j \in V/v_i} d(v_i, v_j)}$$



# Centralidade de Proximidade



$C_p = 0,31$



$C_p = 0,44$

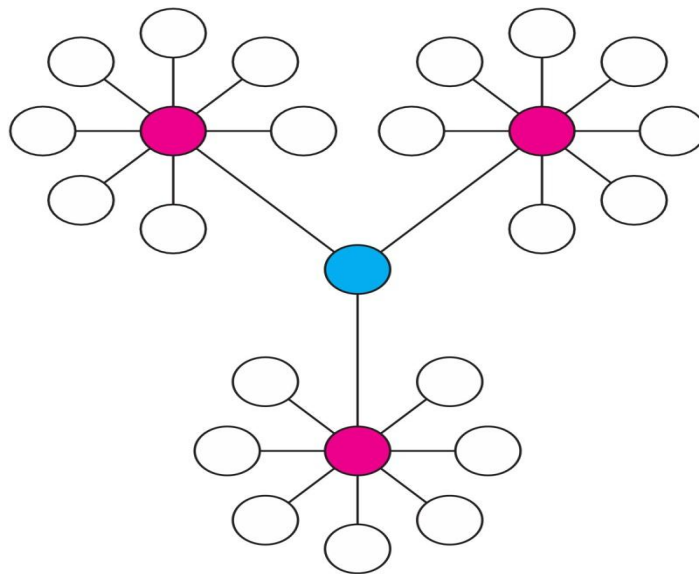


$C_p = 0,53$





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# Betweenness

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# Centralidade de Betweenness

Essa medida foi introduzida para verificar o controle/influência de uma pessoa para transmitir uma informação entre grupos sociais diferentes.

Freeman, Linton (1977). "A set of measures of centrality based upon betweenness". *Sociometry* **40**: 35–41.





# Centralidade de Betweenness

$$C_b(v_i) = \sum_{v_j \neq v_i \neq v_k} \frac{\sigma_{v_j, v_k}(v_i)}{\sigma_{v_j, v_k}}$$



# Centralidade de Betweenness

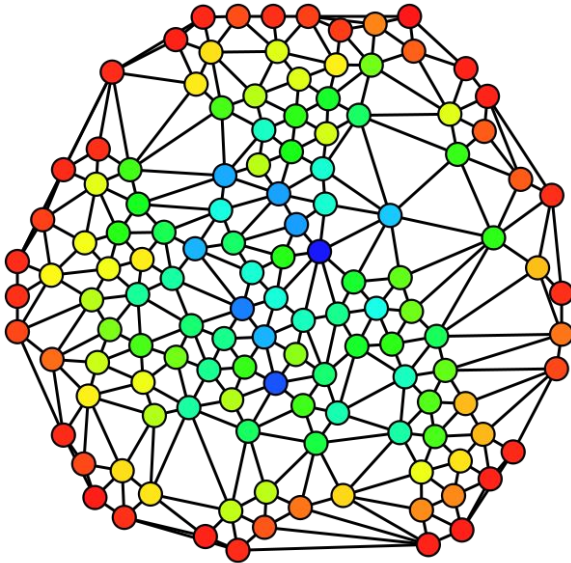
$$Cb'(v_i) = \frac{Cb(v_i)}{(n-1)(n-2)}$$

← Medida normalizada para comparação entre redes

$(n-1)(n-2)$  ← Grau de betweenness do nó mais central em uma rede em formato estrela. Se não for direcionada, dividir por 2



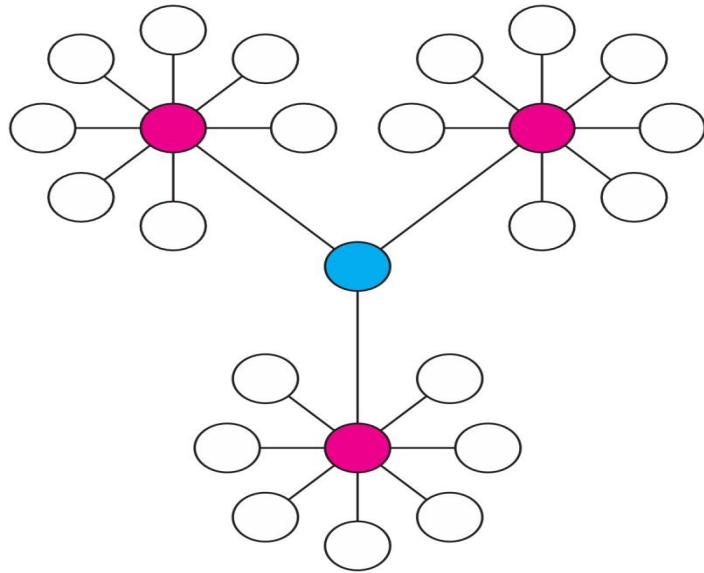
# Centralidade de Betweenness



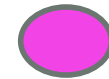
Em redes com **ORDEM** e **TAMANHO** elevados, o valor dessa centralidade é estimado utilizando uma amostragem dos caminhos.



# Centralidade de Betweenness



$C_b = 0,00$



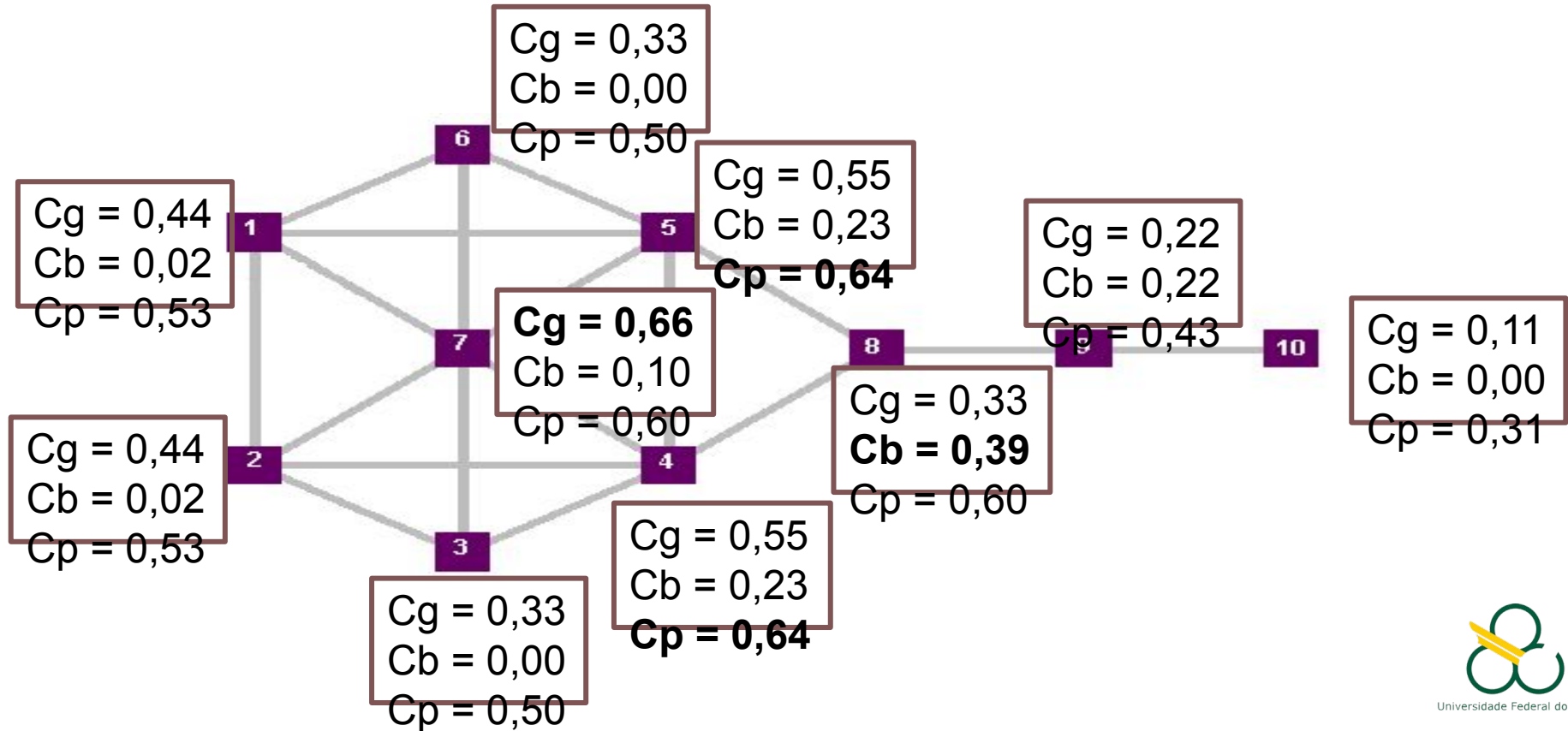
$C_b = 0,51$



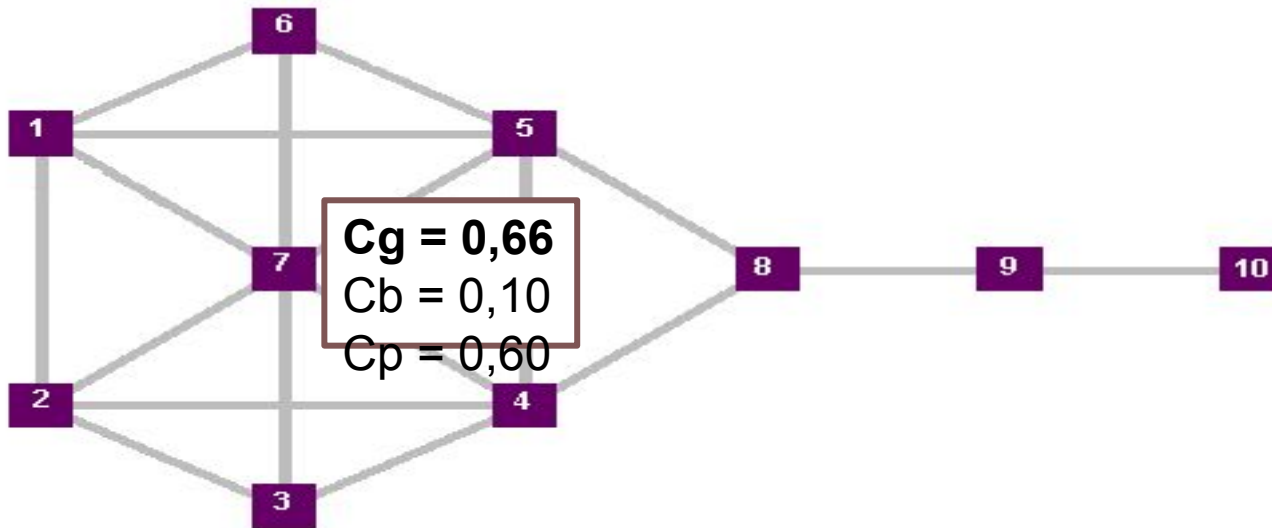
$C_b = 0,70$



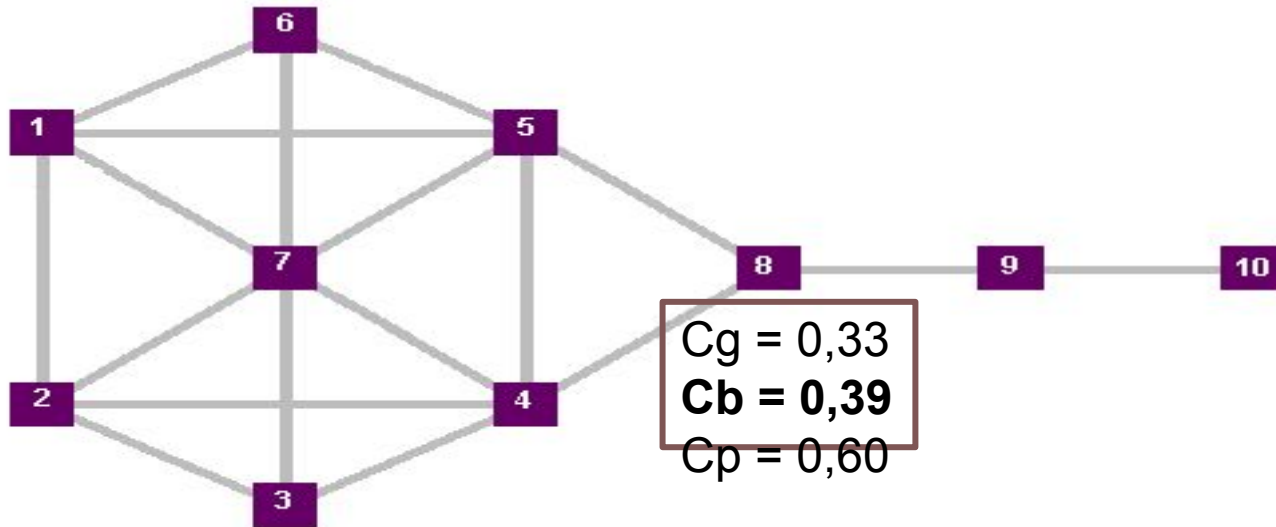
# Centralidade até agora



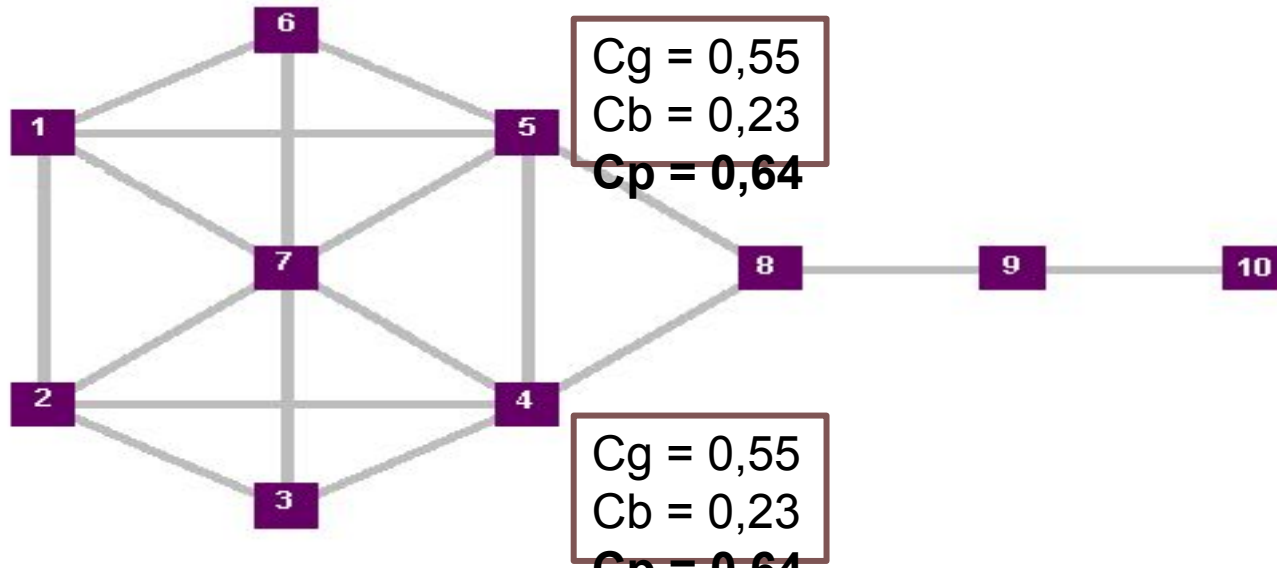
# Centralidade até agora



# Centralidade até agora



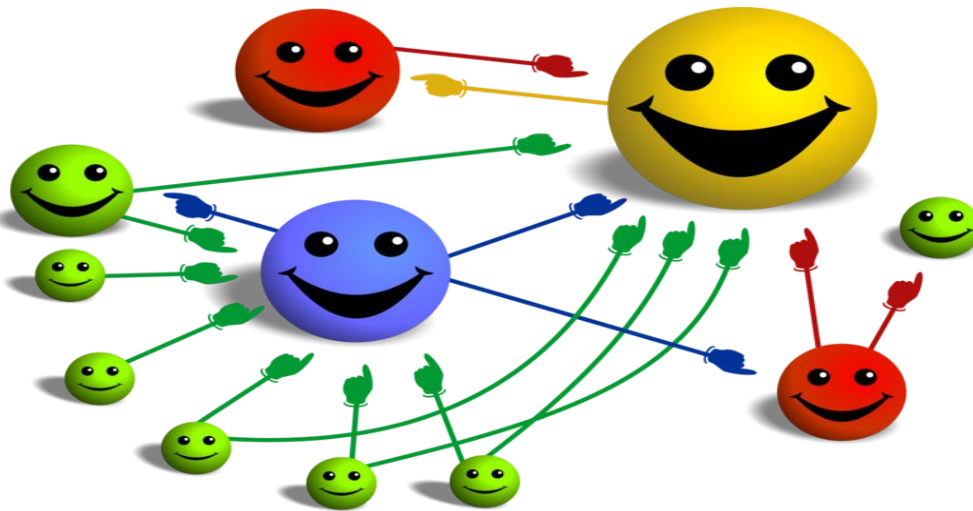
# Centralidade até agora







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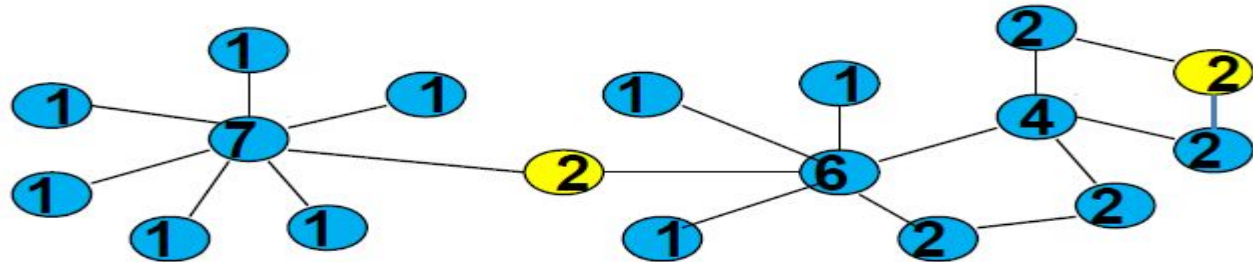
# PageRank

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# Centralidade de Autovetor

Porém, analisemos o seguinte caso:



# Centralidade de Autovetor

$$C_i = \frac{1}{\lambda} \sum_j a_{i,j} C_j$$



# Centralidade de Autovetor

$$\lambda C = \sum_i \sum_j a_{i,j} C_j = AC$$

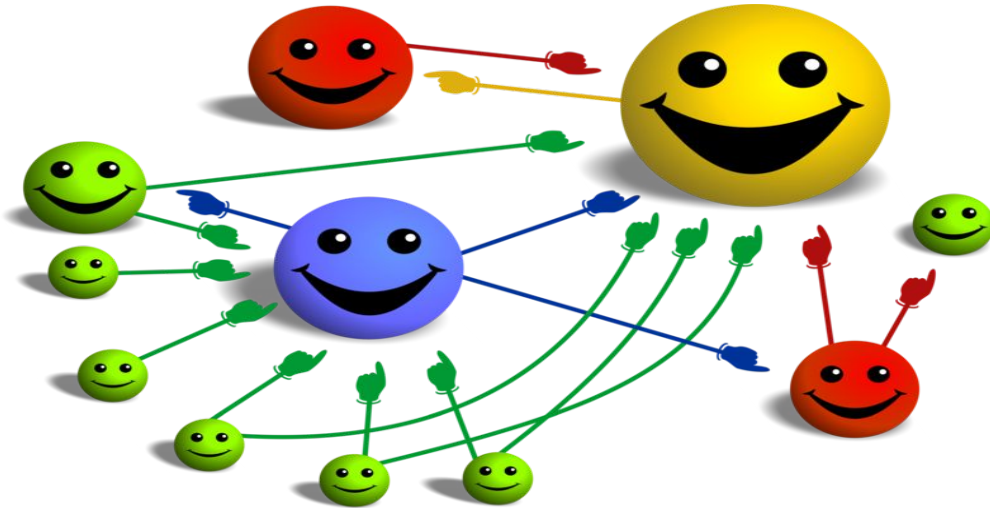


# Autovalor e Autovetor

$$Ax = \lambda x$$



# PageRank



# PageRank

$$C_{pr}(v_i) = \sum_{v_j | (v_j, v_i) \in E} \frac{C_{pr}(v_j)}{dg_{out}(v_j)}$$



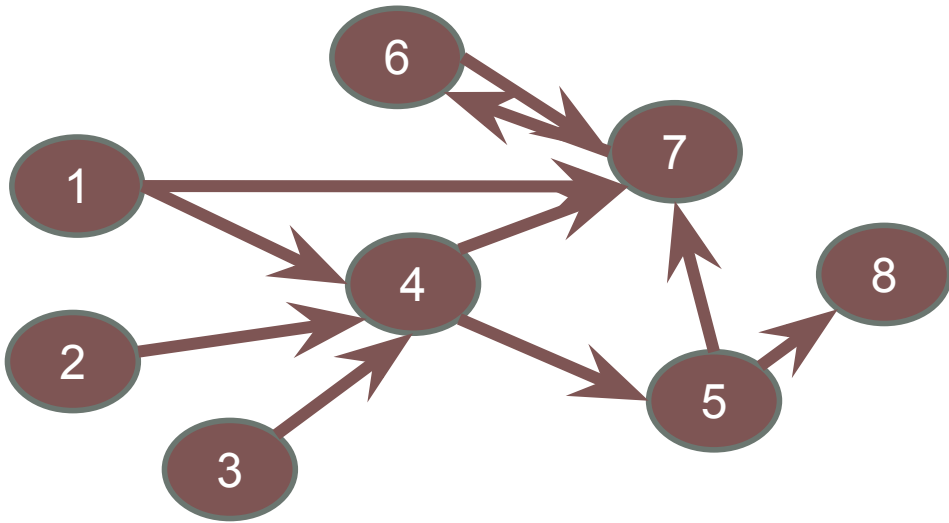
# PageRank

$$C_{pr} = C_{pr}N$$





# PageRank



			0,5			0,5	
			1				
			1				
				0,5		0,5	
						0,5	0,5
						1	
					1		

1	1	1	1	1	1	1	1
---	---	---	---	---	---	---	---



# PageRank

1	1	1	1	1	1	1	1
---	---	---	---	---	---	---	---

X

			0,5			0,5	
			1				
			1				
				0,5		0,5	
						0,5	0,5
						1	
					1		

=

0	0	0	2,5	0,5	1	2,5	0,5
---	---	---	-----	-----	---	-----	-----



# PageRank

0	0	0	2,5	0,5	1	2,5	0,5
---	---	---	-----	-----	---	-----	-----

X

			0,5			0,5	
			1				
			1				
				0,5		0,5	
						0,5	0,5
						1	
					1		

=

0	0	0	0	1,25	2,5	2,5	0,25
---	---	---	---	------	-----	-----	------



# PageRank

0	0	0	0	1,25	2,5	2,5	0,25
---	---	---	---	------	-----	-----	------

 X

			0,5			0,5	
			1				
			1				
				0,5		0,5	
						0,5	0,5
						1	
					1		

=

0	0	0	0	0	2,5	3,125	0,625
---	---	---	---	---	-----	-------	-------



# PageRank

0	0	0	0	0	2,5	3,125	0,625
---	---	---	---	---	-----	-------	-------

X

			0,5			0,5	
			1				
			1				
				0,5		0,5	
						0,5	0,5
						1	
					1		

=

0	0	0	0	0	3,125	2,5	0
---	---	---	---	---	-------	-----	---



# PageRank

0	0	0	0	0	3,125	2,5	0
---	---	---	---	---	-------	-----	---

X

			0,5			0,5	
			1				
			1				
				0,5		0,5	
						0,5	0,5
						1	
					1		

=

0	0	0	0	0	2,5	3,125	0
---	---	---	---	---	-----	-------	---



# PageRank

0	0	0	0	0	2,5	3,125	0
---	---	---	---	---	-----	-------	---

X

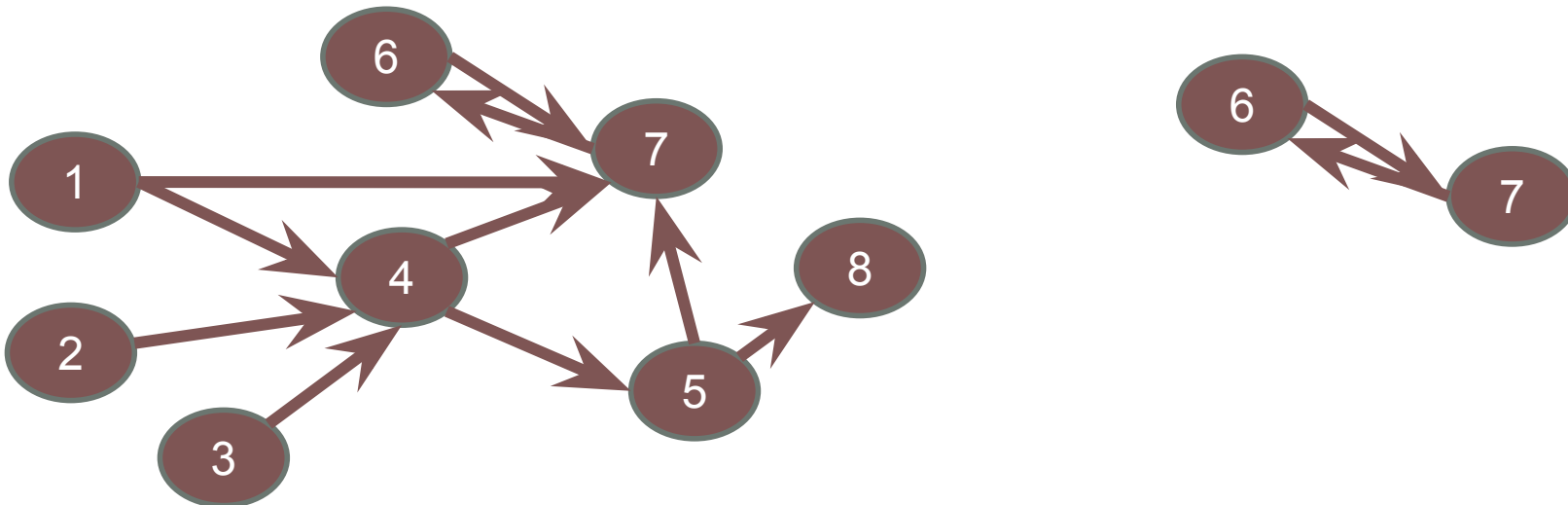
			0,5			0,5	
			1				
			1				
				0,5		0,5	
						0,5	0,5
						1	
					1		

=

0	0	0	0	0	3,125	2,5	0
---	---	---	---	---	-------	-----	---



# PageRank



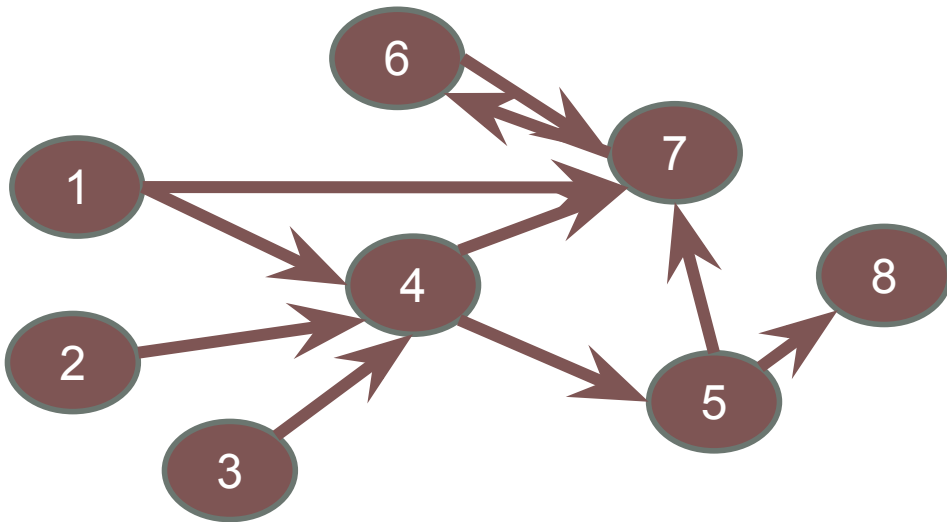


# PageRank

$$P = \alpha N + (1 - \alpha)E$$



# PageRank



0.02	0.02	0.02	0.44	0.02	0.02	0.44	0.02
0.02	0.02	0.02	0.87	0.02	0.02	0.02	0.02
0.02	0.02	0.02	0.87	0.02	0.02	0.02	0.02
0.02	0.02	0.02	0.02	0.44	0.02	0.44	0.02
0.02	0.02	0.02	0.02	0.02	0.02	0.44	0.44
0.02	0.02	0.02	0.02	0.02	0.02	0.87	0.02
0.02	0.02	0.02	0.02	0.02	0.87	0.02	0.02
0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02



# PageRank

$$C_{pr} = \frac{C_{pr}P}{\|C_{pr}P\|}$$



# PageRank

0.04 0.04 0.04 0.65 0.17 0.29 0.65 0.17

0.04 0.04 0.04 0.14 0.33 0.62 0.68 0.11

0.04 0.04 0.04 0.12 0.09 0.6 0.76 0.17

0.04 0.04 0.04 0.12 0.09 0.71 0.68 0.08

0.03 0.03 0.03 0.11 0.09 0.63 0.76 0.07

0.03 0.03 0.03 0.11 0.08 0.71 0.69 0.07

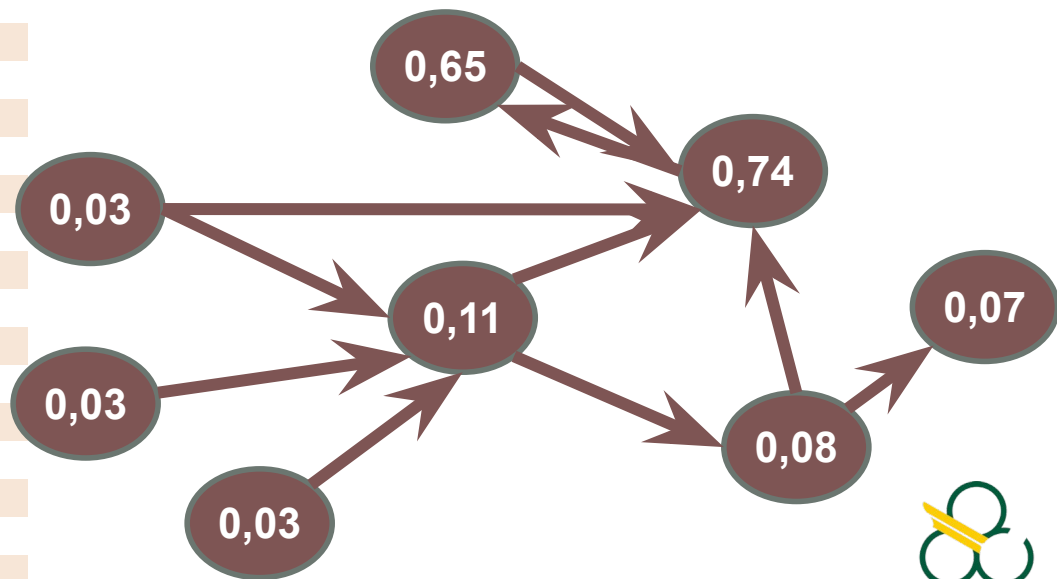
0.03 0.03 0.03 0.11 0.08 0.64 0.75 0.07

0.03 0.03 0.03 0.11 0.08 0.7 0.7 0.07

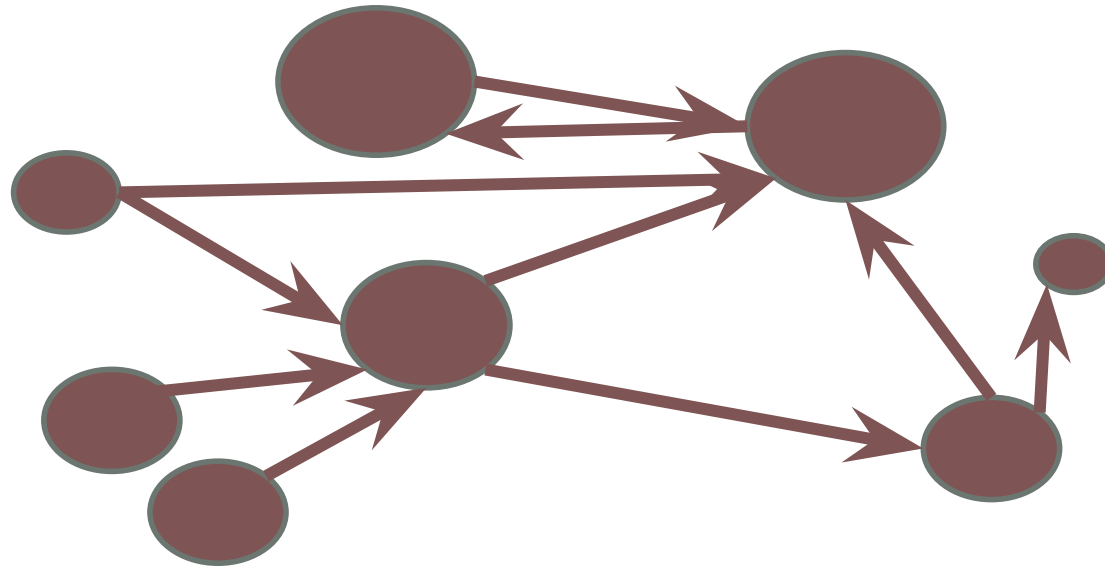
0.03 0.03 0.03 0.11 0.08 0.65 0.75 0.07

0.03 0.03 0.03 0.11 0.08 0.69 0.7 0.07

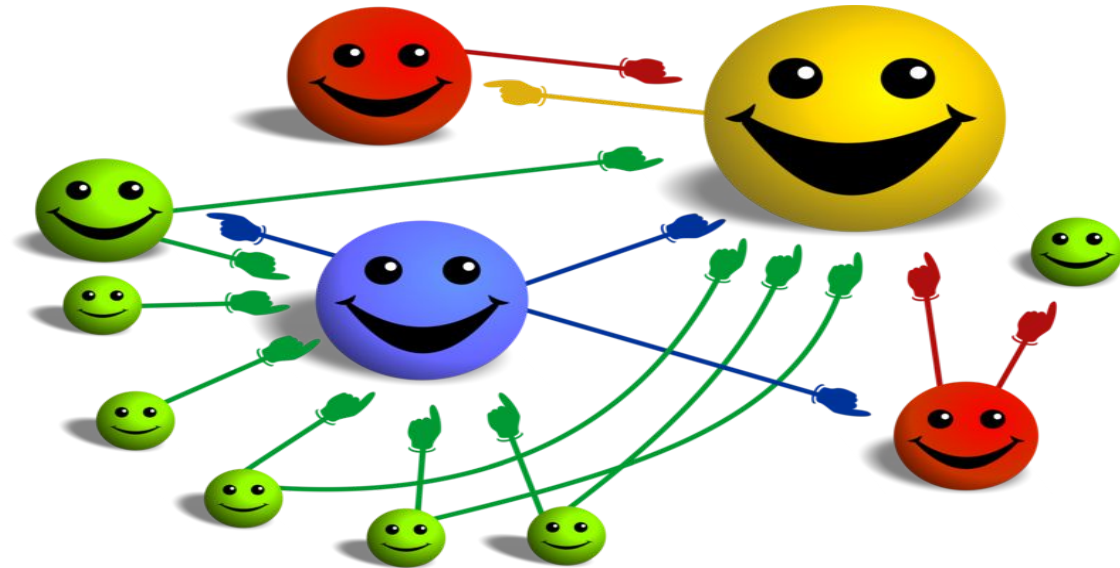
0.03 0.03 0.03 0.11 0.08 0.65 0.74 0.07

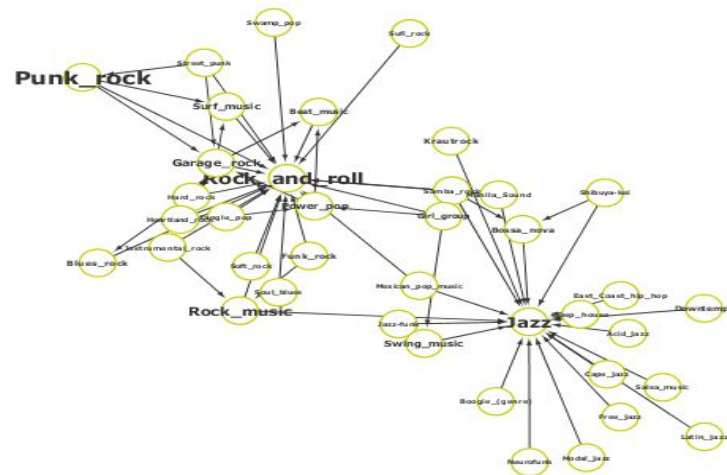


# PageRank



# PageRank

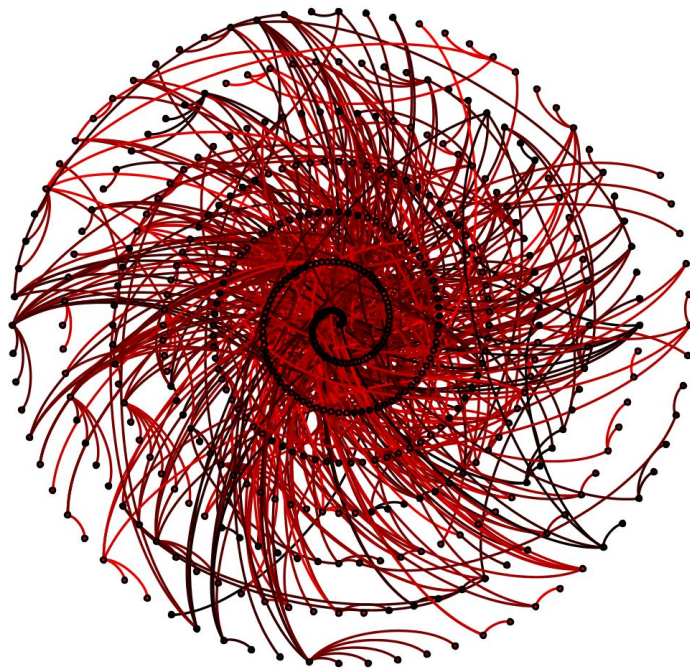




# CENTRALIDADE NAS REDES REAIS

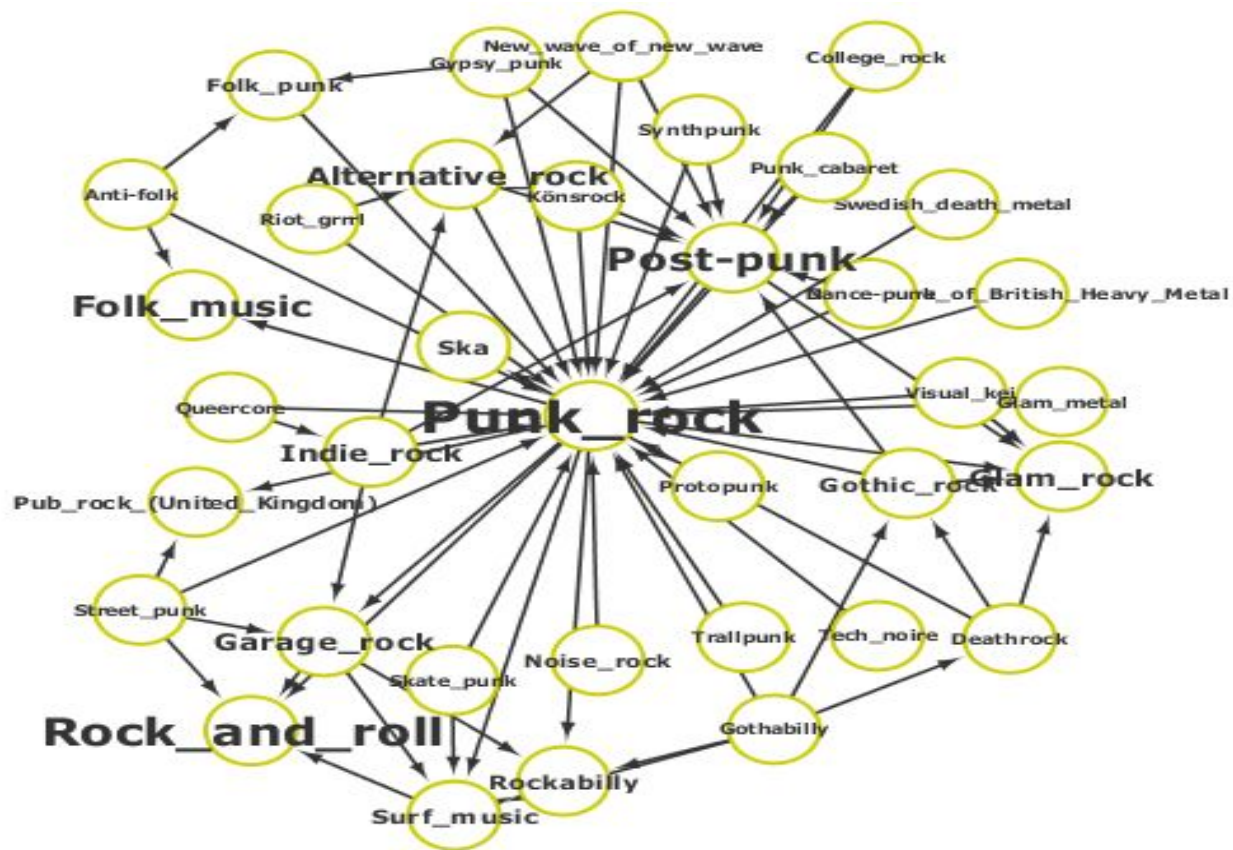
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# Influências Musicais

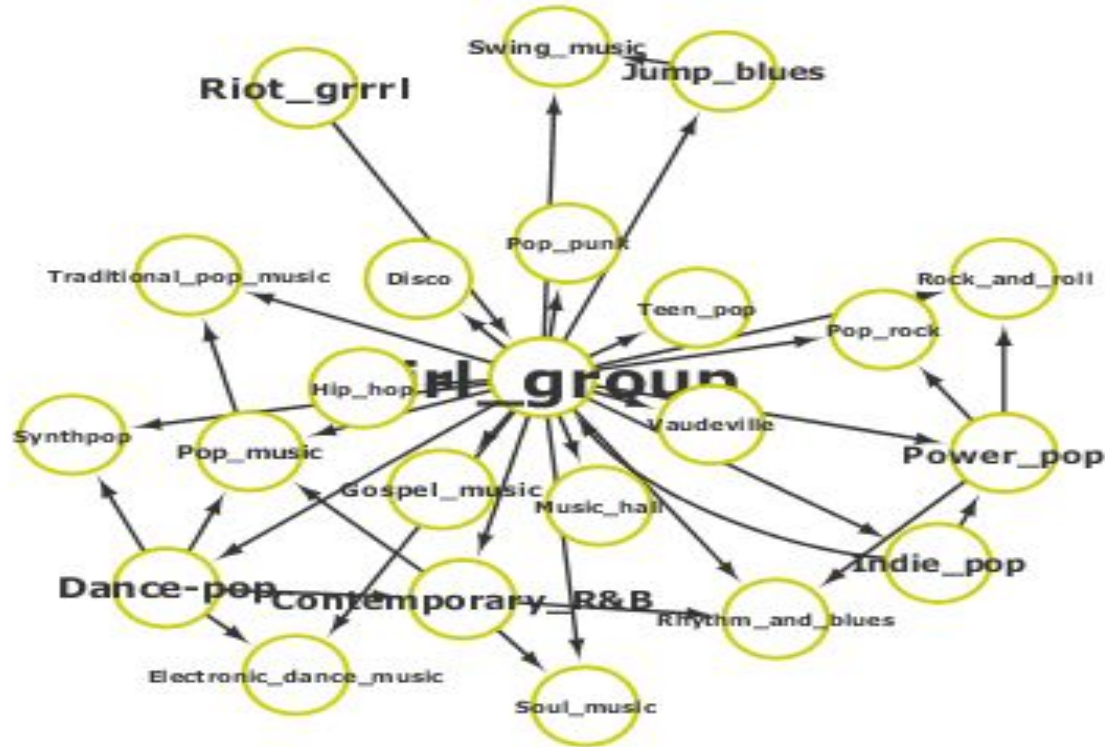




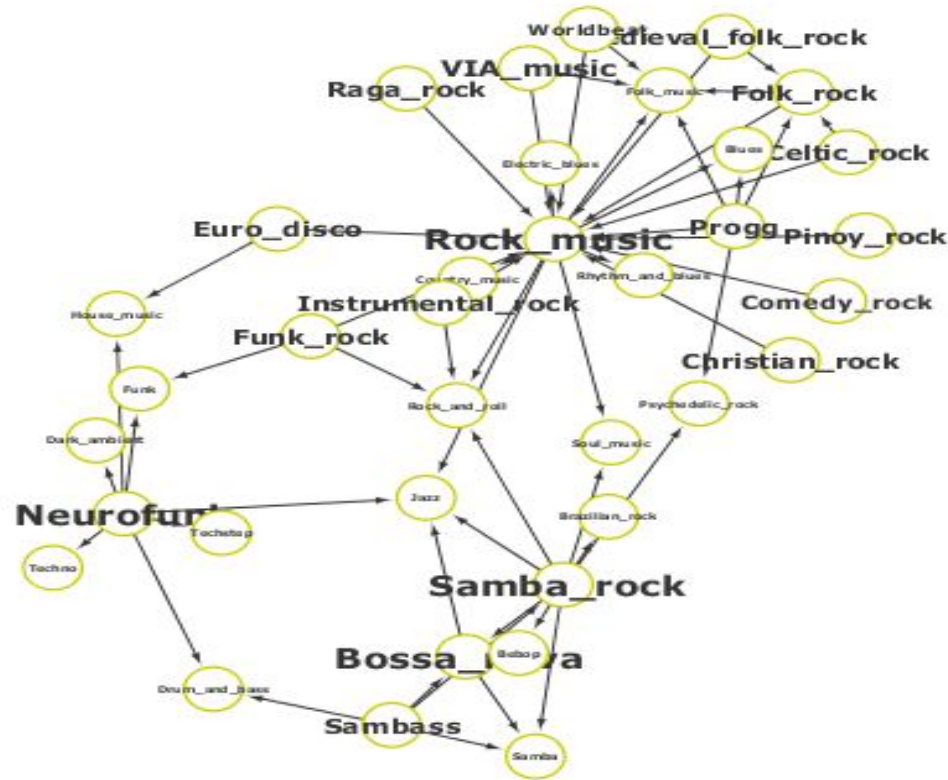
# Influências Musicais – Grau Entrada



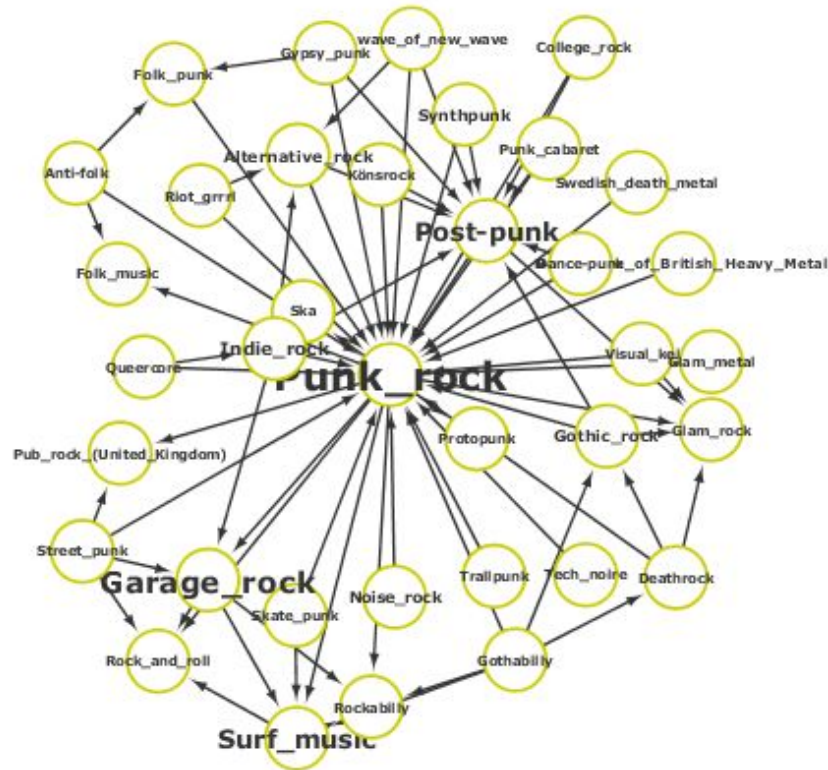
# Influências Musicais – Grau Saída



# Influências Musicais – Proximidade



# Influências Musicais – Betweenness



# Influências Musicais – Pagerank

